3rd Semester

INSTRUMENTATION AND CONTROL ENGINEERING

SUBJECT: BASIC OF CONTROL SYSTEM

SUBJECT CODE : 181531

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Chapter 1 (Basics of control system)

Basics elements of control system.

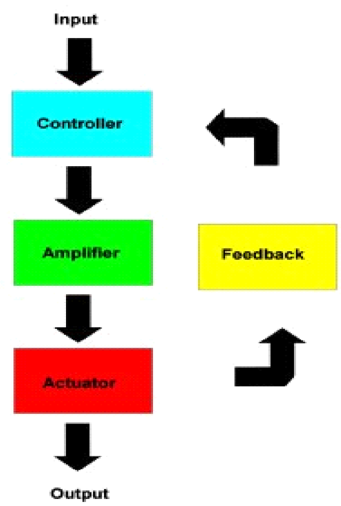
Basic Elements of a Control System. There are four basic elements of a typical motion control system. These are the controller, amplifier, actuator, and feedback. The complexity of each of these elements will vary depending on the types of applications for which they are designed and built.

A controller is the most important component of the control system. It is responsible for the performance of the control system. It is a device or an algorithm that works to maintain the value of the controlled variable at set point. It is responsible for the performance of the control system.

An apparatus for controlling the gain and phase of an input signal input to a power amplifier comprises a gain control loop configured to control the gain of the input signal based on power levels of the input signal and an amplified signal output by the power amplifier, to obtain a predetermined gain of the amplified

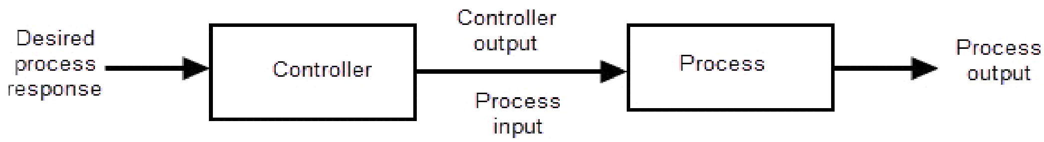
An actuator is a component of a machine that is responsible for moving and controlling a mechanism or system, for example by opening a valve. In simple terms, it is a "mover". When it receives a control signal, an actuator responds by converting the source's energy into mechanical motion.

A feedback control system is a system whose output is controlled using its measurement as a feedback signal. This feedback signal is compared with a reference signal to generate an error signal which is filtered by a controller to produce the system's control input.



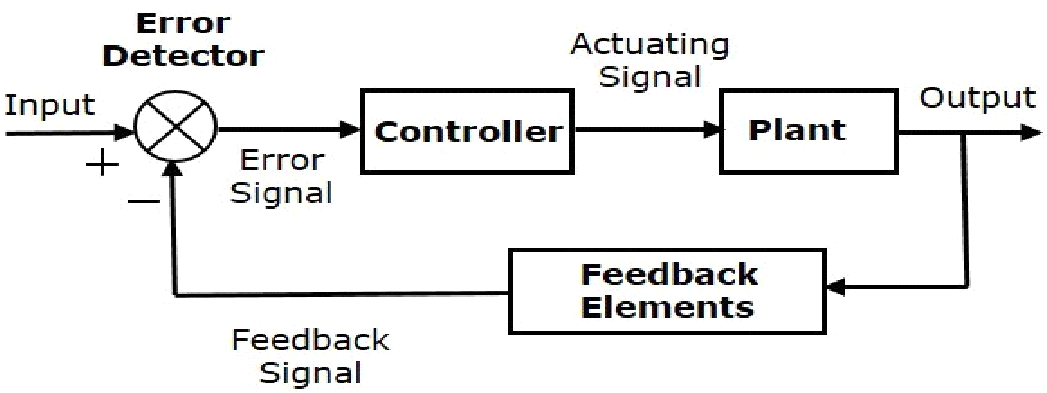
Open loop control system

Then an Open-loop system, also referred to as non-feedback system, is a type of continuous control system in which the output has no influence or effect on the control action of the input signal. In other words, in an open-loop control system the output is neither measured nor "fed back" for comparison with the input.



Closed loop control system

A closed loop control system is a set of mechanical or electronic devices that automatically regulates a process variable to a desired state or set point without human interaction. Closed loop control systems contrast with open loop control systems, which require manual input.



Control System Terminology:

Desired Response – the idealized instantaneous behavior that we would like from the system.

Transient Response – the gradual change in the system as it approaches its approximation of the desired response.

Steady-State Response – the response of the system once it has finished changing and is now approximating the desired response.

Error – the difference between the input and the output of the system.

Steady-State Error – the difference between the steady-state response and the desired response. Stability – the ability of the system to settle into a steady-state response.

Controller – the part of the system that generates the input to the plant or process being controlled.

Open-Loop – a system that does not monitor its output. Open-loop systems can not correct for disturbances.

Closed-Loop – a system that monitors its output and makes corrections to reduce error. By monitoring the output the system can correct for disturbances.

Disturbance – a signal that is not modeled or calibrated in the system leading to corruption of the expected behavior.

Compensator – a system inserted into the controller to improve performance.

Feedback – a path that allows signals from the output of some sub-system to flow back and affect the input of some sub-system earlier in the system signal path.

Robust – a system that will still work as expected with changes to the system parameters, as might be caused by wear of components, or a change in behavior with temperature

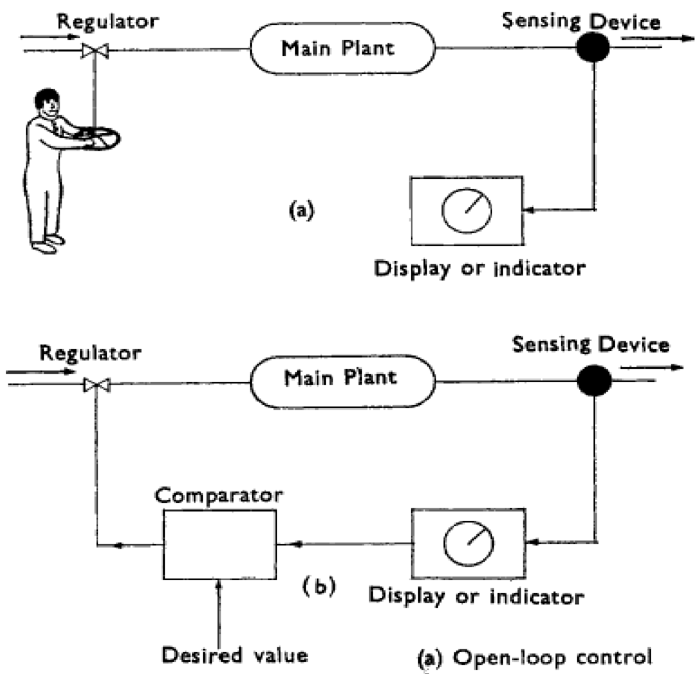
Manually controlled closed loop system.

Closed loop system is to control a process. Let me go with an example.

You have oil in a tank to be heated by hot water. Temperature of oil has to be maintained. Hot water tubes pass through the liquid. Controlling the flow you can heat the oil.

You have to measure the temperature. Feed it to a controller which has a set point defined and with its output, controls a flow control valve, which regulates the hot water. This is a simple closed loop. You don't need to care. This is automatic.

In case there is a disturbance in water or the flow line you can take the AUTO mode to MANUAL, adjust valve position to your requirement. This is an example of manually controlled closed loop. Below figure shows the manually control closed loop system and also shows how automatic works with the help of comparator with setting desired value.



(a) Open-loop control
(b) Closed-loop control

Basics elements of servomechanism:

In control engineering a servomechanism, sometimes shortened to servo, is an automatic device that uses error-sensing negative feedback to correct the action of a mechanism. It usually includes a built-in encoder or other position feedback mechanism to ensure the output is achieving the desired effect. The term correctly applies only to systems where the feedback or error-correction signals help control mechanical position, speed or other parameters.

A servo system primarily consists of three basic components – a controlled device, a output sensor, a feedback system. This is an automatic closed loop control system.

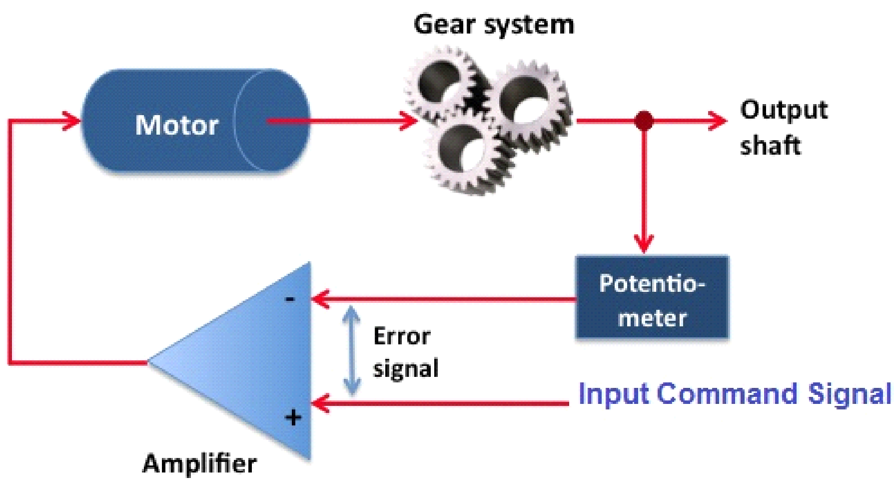
Components of Servomechanism:

A servo system mainly consists of three basic components

- A controlled device
- A output sensor
- A feedback system

Working of Servomechanism:

Servomechanism is an automatic closed loop control system. Here instead of controlling a device by applying variable input signal, the device is controlled by a feedback signal generated by comparing output signal and reference input signal.



Carefully observe the figure above and think. When reference input signal or command signal is applied to the system, it is compared with output reference signal of the system produced by output sensor, and a third signal produced by feedback system. This third signal acts as input signal of controlled device. This input signal to the device presents as long as there is a logical difference between reference input signal and output signal of the system. After the device achieves its desired output, there will be no longer logical difference between reference input signal and reference output signal of the system. Then, third signal produced by comparing theses above said signals will not remain enough to operate the device further and to produce further output of the system until the next reference input signal or command signal is applied to the system.

Hence the primary task of a servomechanism is to maintain the output of a system at the desired value in the presence of disturbances.

Examples of automatic control system: Here are the some examples for automatic control system for day to day life.

- **Air conditioner**
- **Rice cooker**
- **Automatic ticketing machine**
- **Refrigerators**
- **vacuum cleaner**

linear and nonlinear system:

Linear system: linear systems are those systems which follow the principle of superposition and principle of homogeneity.

Nonlinear system: Nonlinear systems are those systems which do not follow the principle of superposition and principle of homogeneity.

Difference between linear and nonlinear control system.

Linear system	Nonlinear system
1. which follow the principle of superposition and homogeneity.	1.which don't follow the principle of superposition and homogeneity.
2. can be analyses be standard test signals	2. cannot be analyses be standard test signals
3. Stability depends only on root locations.	3. stability depends only on root locations ,initial conditions and type of inputs
4. Do not exhibit the limit cycle.	4. exhibit limit cycle.
5. do not exhibit the hysteresis/jump response	5. exhibit the hysteresis/jump response
6. can be analyzed by Laplace transform and z- transform	6. cannot be analyzed by Laplace transform and z- transform

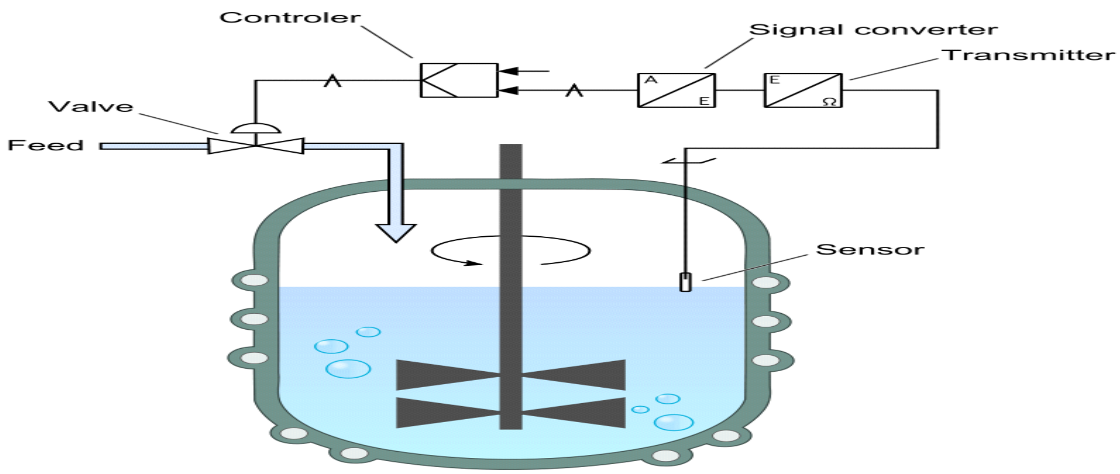
Control system examples: here we discuss the various examples of control systems.

1. Chemical systems:

The continuous stirred-tank reactor (CSTR), also known as vat- or backmix reactor, or a continuous-*flow* stirred-tank reactor (C*F*STR), is a common model for a chemical reactor in chemical engineering and environmental engineering. A CSTR often refers to a model used to estimate the key unit operation variables when using a continuous agitated-tank reactor to reach a specified output. The mathematical model works for all fluids: liquids, gases, and slurries.

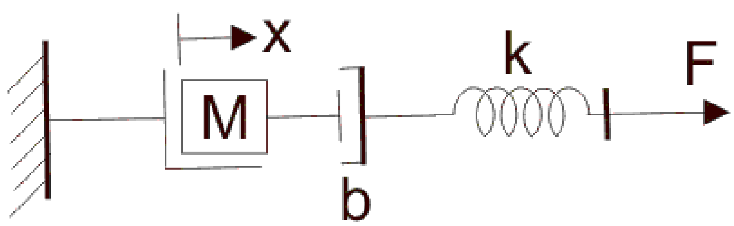
Within the field of environmental engineering, CSTRs are usually applied in wastewater treatment processes. CSTRs are commonly used in the activated sludge treatment process during wastewater treatment. Anaerobic digesters for the stabilization of biosolids produced during biological treatment of wastewater are also designed as CSTRs.

CSTRs facilitate rapid dilution rates which make them resistant to both high pH and low pH volatile fatty acid wastes. CSTRs are less efficient compared to other types of reactors as they require larger reactor volumes to achieve the same reaction rate as other reactor models such as Plug Flow Reactors (PFR).



Mechanical system:

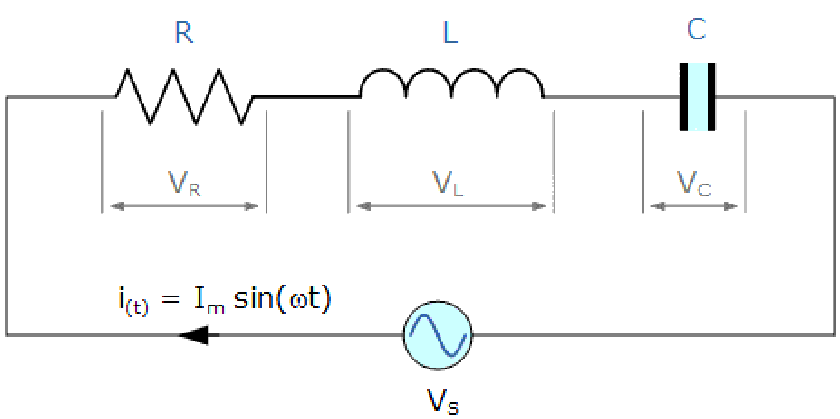
Mechanical systems obey Newton's law that the sum of the forces equals zero; that is, the sum of the applied forces must be equal to the sum of the reactive forces. The three qualities characterizing elements in a mechanical translation* system are mass, elastance, and damping.



Electrical system:

below shown is the example of electrical system.

v-ri(t)-Ldi(t)/dt-1/c∫dt=0



Introduction to Laplace transform:

The Laplace transform method is used extensively to facilitate and Systematize the solution of ordinary constant-coefficient differential Equations. The advantages of this modern transform method for the analysis of linear-time-invariant (LTI) systems are the following:

1. It includes the boundary or initial conditions.
2. The work involved in the solution is simple algebra.
3. The work is systematized.
4. The use of a table of transforms reduces the labor required.
5. Discontinuous inputs can be treated.
6. The transient and steady-state components of the solution are Obtained simultaneously. The disadvantage of transform methods is that if they are used Mechanically, without knowledge of the actual theory involved, they some- times yield erroneous results. Also, a particular equation can sometimes be solved more simply and with less work by the classical method. Although an understanding of the Laplace transform method is essential, it must be emphasized that the solutions of differential equations are readily obtained by use of CAD packages such as MATLAB and TOTAL-PC

Chapter 2 BCS

Control Components

Servomotor

In simple words, the **servo motor** is an individual electric motor. The main purpose of using this motor in industries is to rotate and push the machine parts wherein, as well as task must be defined. This is one of the most extensively used motors in applications like industrial manufacture, automation development, etc. Even though these are proposed and planned to utilize in the applications like motion control for high precision positioning, rapid reversing and outstanding performance. The **applications of servomotors** include in robotics, automated industrialized systems, radar systems, tracking systems, machine apparatus, computers, CNC machines, etc.

Another name of the servo motor is controlled motor, because these are employed in feedback control systems like output actuators & doesn't utilize for continuous energy alteration. The **working principle of the servomotor** and electromagnetic motor are same except the structure and the function are dissimilar. The power rating of these motors will changes from a watt to a few hundred watts. When the rotor inactivity in the motor is small then the response will be high. Similarly, the rotor of the servomotor is high then the motor has a lesser diameter. They function at less speed or zero speed sometimes. Servo motors are applicable in radar, machine tool, computers, robot, tracking, processing controlling, guidance systems, etc.

Different Types of Servo Motors

Generally, these motors are categorized into two types based on the supply used for its function such as **AC servo motors** & **DC servo motors**. These motors are suitable for many applications due to the progress of **the microprocessor**, power transistor & high precision control. These motors include three wires power, ground, and control. Based on the size and outline, these motors are used in various applications. The most frequently used servo motor is RC servo motor which is mostly used in hobby applications. The main features of this motor include affordability, simplicity, and consistency.

DC Servo Motor

Generally, the DC servo motor includes a DC source separately in the field of the armature winding. The motor can be controlled either by managing the field current otherwise the armature current. The armature control has some benefits compare with field control. Similarly, field control has come benefits compare with armature control. The controlling of this motor can be done based on the application used. This motor offers a quick and accurate response to begin or end command signals because of the small armature inductive reactance. These motors are utilized in several devices and numerically controlled equipment.

Types of DC Servo Motors

The DC servo motors are classified into different types which are

- **Series Motors**
- **Split Series Motors**
- **Shunt Control Motor**
- **Permanent Magnet Shunt Motor**

Series Motors

The series type DC servo motors include high starting torque as well as draws huge current. The speed regulation of this motor is very less. Turnaround can be attained by overturning the field voltage polarity using split series field winding. The efficiency of this motor can be reduced by using this method.

Shunt motor

The shunt control servo motor is not dissimilar from any kind of DC shunt motor. This motor includes two windings such as field windings and armature windings. Field windings are located on the stator of the machine whereas the armature windings are located on the rotor. The connection between these two windings can be done to a DC source. In a DC shunt motor, the two windings are connected across the DC source in parallel.

Permanent Magnet Shunt Motor

This is a permanent excitation motor wherever the field is in fact supplied by a stable magnet. The motor performance is same to armature controlled permanent field motor that we are going to recognize in the next segment.

AC Servo Motor

AC servo motor includes an encoder which is used by the controllers to give the feedback as well as closed loop control. AC motor can be located to high accuracy as well as controlled accurately as necessary for the applications. These motors have superior designs in order to achieve better torque. The AC servo motor applications mainly include in robotics, automation, CNC equipment, and many more applications.

Types of AC Servo Motors

The AC servo motors are classified into different types which are

- **Positional Rotation Servo Motor**
- **Continuous Rotation Servo Motor**
- **Linear Servo Motor**

Positional Rotation Servo Motor

The most common kind of servo motor is Positional rotation motor. The output of the shaft in motor rotates with 180 degrees. This type of motor mainly comprises includes physical stops that are placed in the gear mechanism to prevent rotating outside to protect the rotation sensor. The applications of positional rotation servo motor include in robots, aircraft, toys, controlled cars, & many more applications.

Continuous Rotation Servo Motor

Both common positional rotation servo motor and continuous rotation servo motor are same, except it can go in every direction without a fixed limit. The control signal alternately locates the static point of the servo to understand the direction as well as the speed of rotation. The variety of potential commands will cause the motor for rotating in the directions of clockwise otherwise anticlockwise as chosen by altering speed, based on the control signal. The application of continuous rotation servo motor includes a radar dish. For example, if you are traveling single on a robot otherwise you can employ one like a drive motor over a mobile robot.

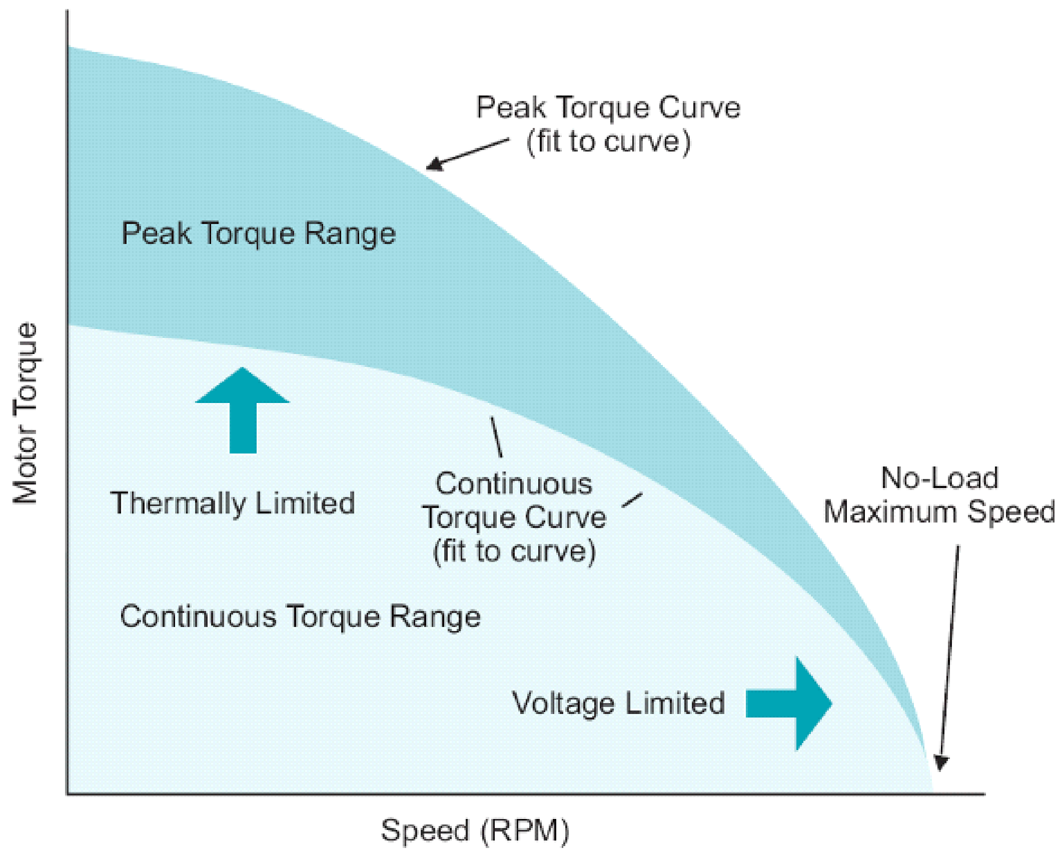
Linear Servo Motor

The Linear servo motor is one kind of motor and it is similar to the positional rotation servo motor, however with extra mechanisms for changing the output from circular in the direction of back-and-forth. We cannot find these motors easily, although occasionally you can discover them at hobbyist stores everywhere they are used like actuators within advanced model airplanes.

Thus, this is all about types of servo motors. This motor is a division of servomechanism and coupled with some type of encoder for providing positioning, speed feedback as well as some fault correcting apparatus which activates the supply signal. The basic characteristics to be required for any servo motor includes, the output torque of the motor must be proportional to the applied voltage. The torque direction which is expanded by the motor must be depending on the instantaneous polarity of the control voltage.

The torque speed characteristics of servo motor :

The torque speed characteristic of an ac servo motor is fairly linear and has negative slope throughout. The rotor construction is usually squirrel cage or drag cup type for an ac servo motor. The diameter is small compared to the length of the rotor which reduces inertia of the moving parts.



Torque speed characteristics of servomotor

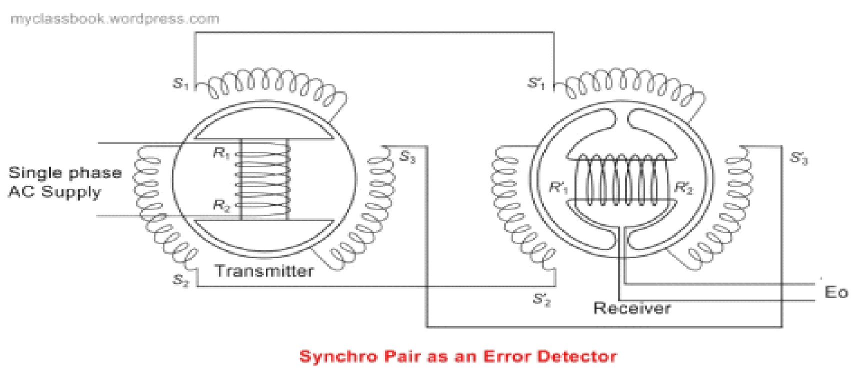
synchro

A **synchro** (also known as **selsyn** and by other brand names) is, in effect, a transformer whose primary-to-secondary coupling may be varied by physically changing the relative orientation of the two windings. Synchros are often used for measuring the angle of a rotating machine such as an antenna platform. In its general physical construction, it is much like an electric motor. The primary winding of the transformer, fixed to the rotor, is excited by an alternating current, which by electromagnetic induction, causes voltages to appear between the Y-connected secondary windings fixed at 120 degrees to each other on the stator. The voltages are measured and used to determine the angle of the rotor relative to the stator.

Synchro pair as an error detector

We can also use the pair of Synchro pair as an error detector. Here error means the output voltage which depends upon the difference between the angular positions of two rotors of synchro pair.
As the name indicates, it uses two synchros. First synchro is called as synchro generator (or transmitter) and second synchro is called as control transformer (or receiver).

Synchro generator has dumb-bell shaped (salient pole) rotor whereas control transformer contains umbrella shaped rotor. For ease of understanding consider that initially two rotors as perpendicular to each other.



Synchro pair as an error detector

When single phase AC supply is applied to the rotor of synchro generator an alternating flux is generated in the rotor and the empty space between rotor and stator. The stator has three windings, one end of each winding is connected in the star connection and ends are connected to the three ends of the stator of control transformer. Remaining three ends of the control transformer are also connected in star fashion.

As alternating flux is generated in the rotor of generator it produces statically induced emf in the stator windings. As these windings are connected to the three windings of the stator of the control transformer same current flows through it.

Initially, the flux axis of both the rotors are perpendicular so that the output voltage (E0) will be zero because it depends upon cos (θ-a). Where θ is angular position of first rotor and a is angular position of the second rotor.

As the angle between two rotor changes output voltage also changes which is given by,

E0 = Eom cos(θ-a) sin(wt-ß)

Where

- θ = angular position of the rotor of the generator,
- a = angular position of the rotor of the control transformer,
- ß = phase lag due to resistances and inductances of the windings,

let us define a new angle for ease of calculations as , d = a + 90, a = d – 90

so the output voltage becomes,

$$Eo = Eom \cos[\theta-(d-90)] \sin(wt-\beta)$$

$$Eo = Eom \cos[90+(\theta-d)] \sin(wt-\beta)$$

$$Eo = Eom [-\sin(\theta-d)] \sin(wt-\beta) \qquad (\text{since } \cos(90+A) = -\sin(A))$$

$$Eo = Eom \sin(d-\theta) \sin(wt-\beta)$$

For small value of $(d-\theta)$, $\sin(d-\theta) \approx (d-\theta)$

Therefore

$$E = Eom (d-\theta) \sin(wt-\beta)$$

Above equation gives the value of error voltage. Thus the synchro pair can be used as an error detector.

Tachometer

A tachometer (revolution-counter, tach, rev-counter, RPM gauge) is an instrument measuring the rotation speed of a shaft or disk, as in a motor or other machine. The device usually displays the revolutions per minute (RPM) on a calibrated analogue dial, but digital displays are increasingly common.

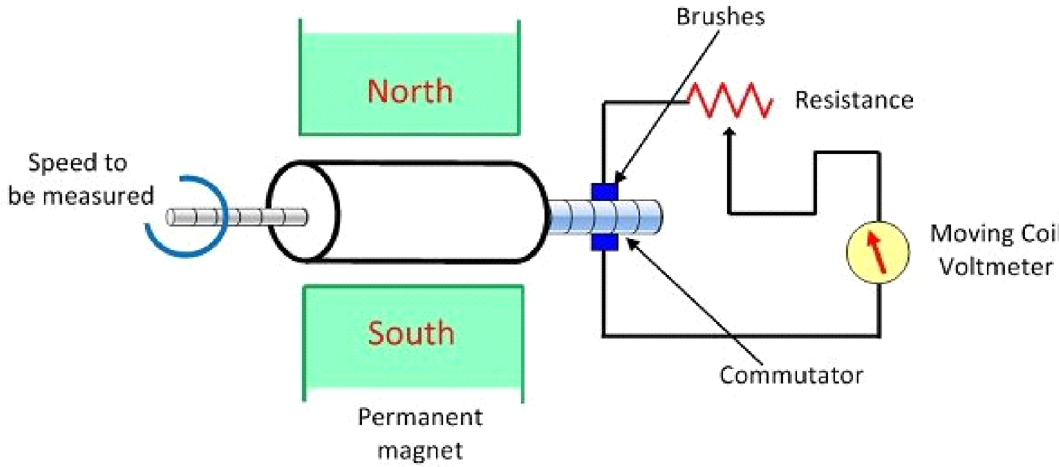
Tachometer is an instrument used for measuring the rotation or revolution speed of objects, such as an engine or a shaft. The tachometer measures revolutions per minute (RPMs) of engines and is widely used in automobiles, airplanes, marine engineering field and many others.

The operation of the tachometer generator is based on the principle that the angular velocity of rotor is proportional to the generated EMF if the excitation flux is constant. These tachometers are specified with generated voltage, accuracy, maximum speed, ripples and operating temperature.

DC Tachometer Generator

Permanent magnet, armature, commutator, brushes, variable resistor, and the moving coil voltmeter are the main parts of the DC tachometer generator. The machine whose speed is to be measured is coupled with the shaft of the DC tachometer generator.

The DC tachometer works on the principle that when the closed conductor moves in the magnetic field, EMF induces in the conductor. The magnitude of the induces emf depends on the flux link with the conductor and the speed of the shaft.



DC Tachometer Generator

The armature of the DC generator revolves between the constant field of the permanent magnet. The rotation induces the emf in the coil. The magnitude of the induced emf is proportional to the shaft speed.

The commutator converts the alternating current of the armature coil to the direct current with the help of the brushes. The moving coil voltmeter measures the induced emf. The polarity of the induces voltage determines the direction of motion of the shaft. The resistance is connected in series with the voltmeter for controlling the heavy current of the armature.

The emf induces in the dc tachometer generator is given as

$$E = \frac{\emptyset PN}{60} \times \frac{z}{a}$$

Where,

E – generated voltage

Φ – flux per poles in Weber

P- number of poles

N – speed in revolution per minutes

Z – the number of the conductor in armature windings.

a – number of the parallel path in the armature windings.

$$E \,\alpha\, N$$

$$E = KN$$

$$K = Constant = \frac{\emptyset P}{60} \times \frac{z}{a}$$

Advantages of the DC Generator

The following are the advantages of the DC Tachometer.

- The polarity of the induces voltages indicates the direction of rotation of the shaft.
- The conventional DC type voltmeter is used for measuring the induces voltage.

Disadvantages of DC Generator

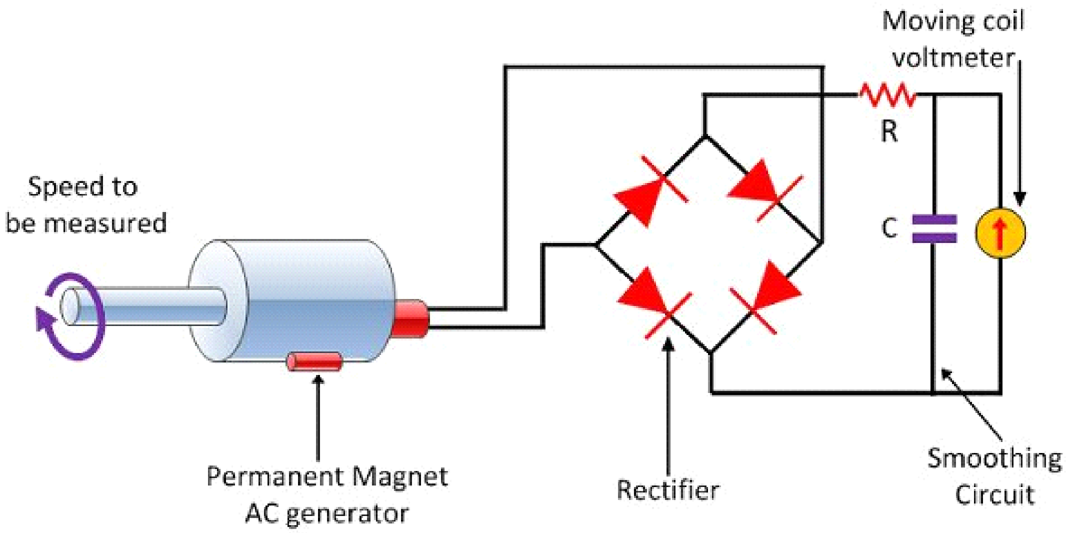
- The commutator and brushes require the periodic maintenance.
- The output resistance of the DC tachometer is kept high as compared to the input resistance. If the large current is induced in the armature conductor, the constant field of the permanent magnet will be distorted.

AC Tachometer Generator

The DC tachometer generator uses the commutator and brushes which have many disadvantages. The AC tachometer generator designs for reducing the problems. The AC tachometer has stationary armature and rotating magnetic field. Thus, the commutator and brushes are absent in AC tachometer generator.

The rotating magnetic field induces the EMF in the stationary coil of the stator. The amplitude and frequency of the induced emf are equivalent to the speed of the shaft. Thus, either amplitude or frequency is used for measuring the angular velocity.

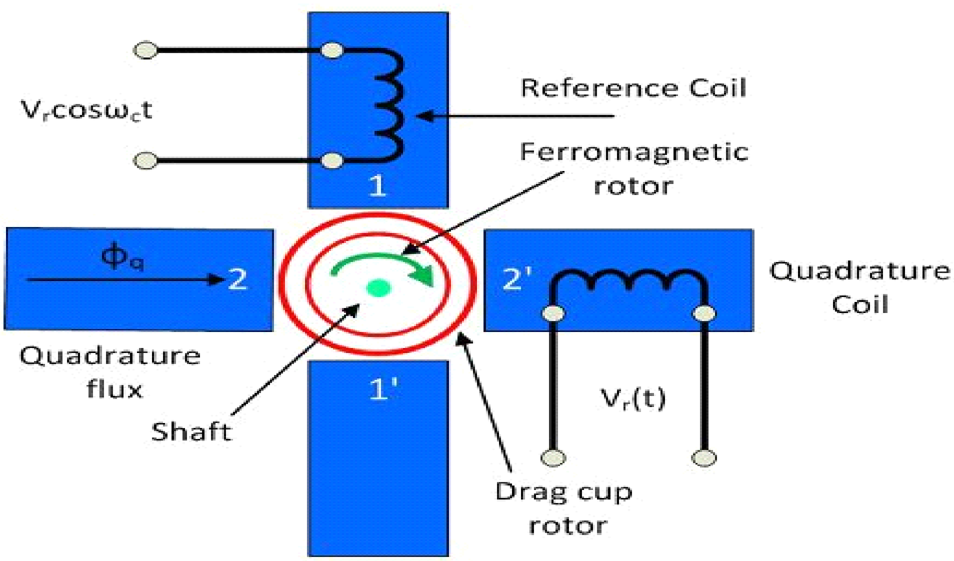
The below mention circuit is used for measuring the speed of the rotor by considering the amplitude of the induced voltage. The induces voltages are rectified and then passes to the capacitor filter for smoothening the ripples of rectified voltages.



A.C Tachometer Generator

Drag Cup Rotor AC Generator

The drag cup type A.C tachometer is shown in the figure below.



A.C Tachometer Generator

The stator of the generator consists two windings, i.e., the reference and quadrature winding. Both the windings are mounted 90° apart from each other. The rotor of the tachometer is made with thin aluminium cup, and it is placed between the field structure.

The rotor is made of the highly inductive material which has low inertia. The input is provided to the reference winding, and the output is obtained from the quadrature winding. The rotation of rotor between the magnetic field induces the voltage in the sensing winding. The induces voltage is proportional to the speed of the rotation.

Advantages

- The drag cup Tachogenerator generates the ripple free output voltage.
- The cost of the generator is also very less.

Disadvantage

The nonlinear relationship obtains between the output voltage and input speed when the rotor rotates at high speed.

Chapter 3 BCS

Control system representation

Transfer function

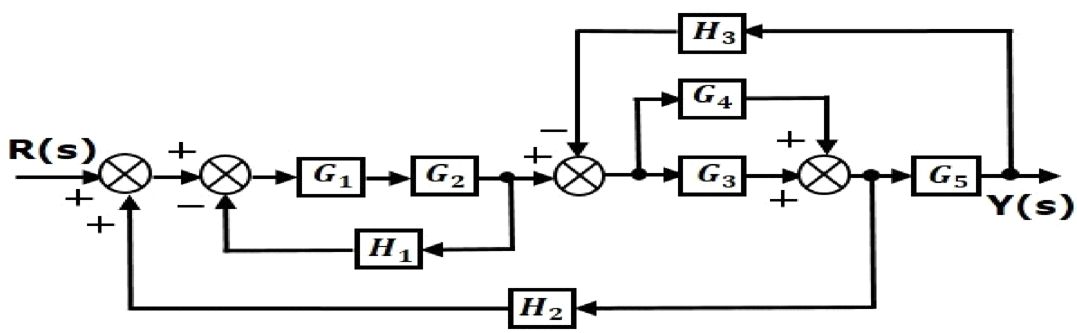
Transfer function is the ratio of Laplace transform of the output signal to Laplace transform of input signal with assumptions that all initial conditions are zero.

Block Diagram Reduction Rules

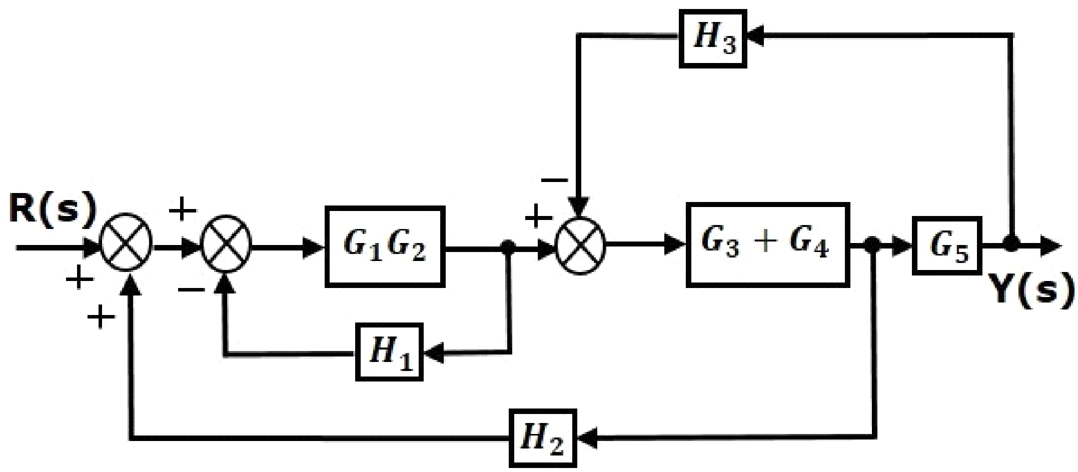
Follow these rules for simplifying (reducing) the block diagram, which is having many blocks, summing points and take-off points.

- **Rule 1** – Check for the blocks connected in series and simplify.
- **Rule 2** – Check for the blocks connected in parallel and simplify.
- **Rule 3** – Check for the blocks connected in feedback loop and simplify.
- **Rule 4** – If there is difficulty with take-off point while simplifying, shift it towards right.
- **Rule 5** – If there is difficulty with summing point while simplifying, shift it towards left.
- **Rule 6** – Repeat the above steps till you get the simplified form, i.e., single block.

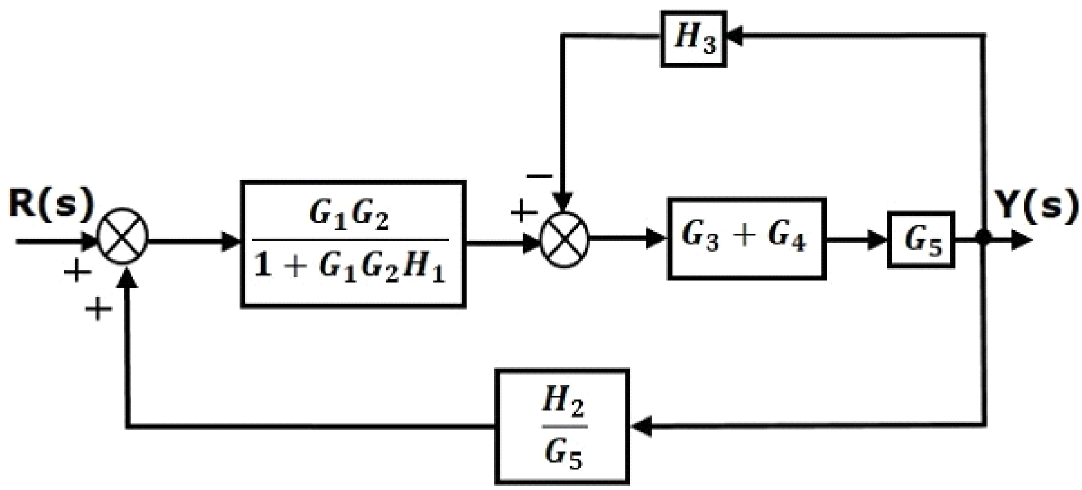
Consider the block diagram shown in the following figure. Let us simplify (reduce) this block diagram using the block diagram reduction rules.



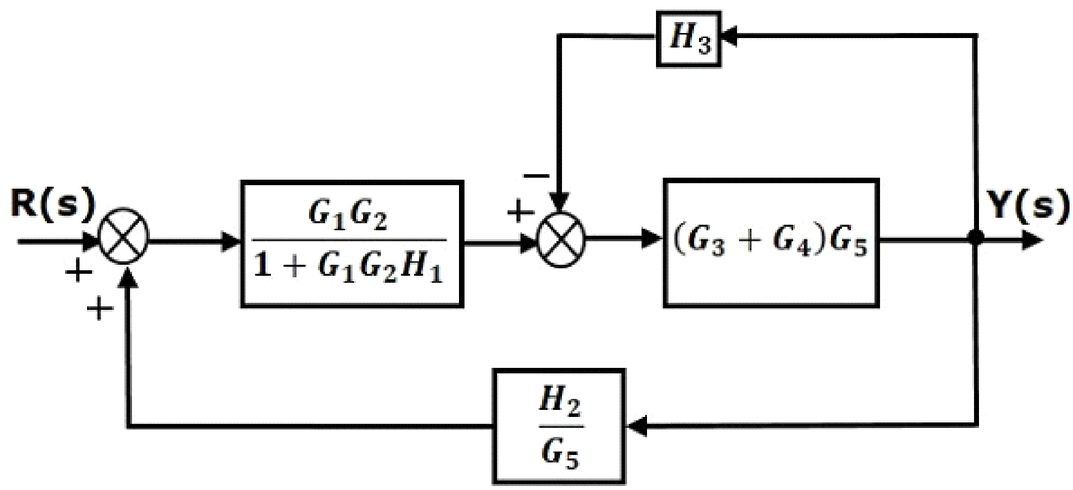
Step 1 – Use Rule 1 for blocks G_1G1 and G_2G2. Use Rule 2 for blocks G_3G3 and G_4G4. The modified block diagram is shown in the following figure.



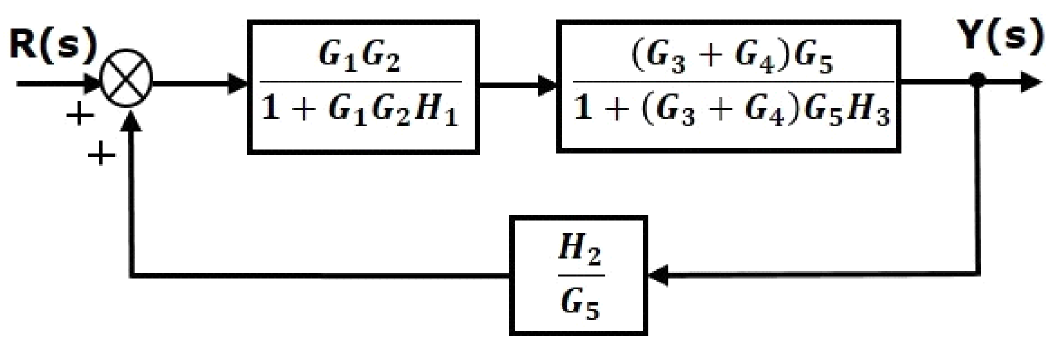
Step 2 – Use Rule 3 for blocks G_1G_2G1G2 and H_1H1. Use Rule 4 for shifting take-off point after the block G_5G5. The modified block diagram is shown in the following figure



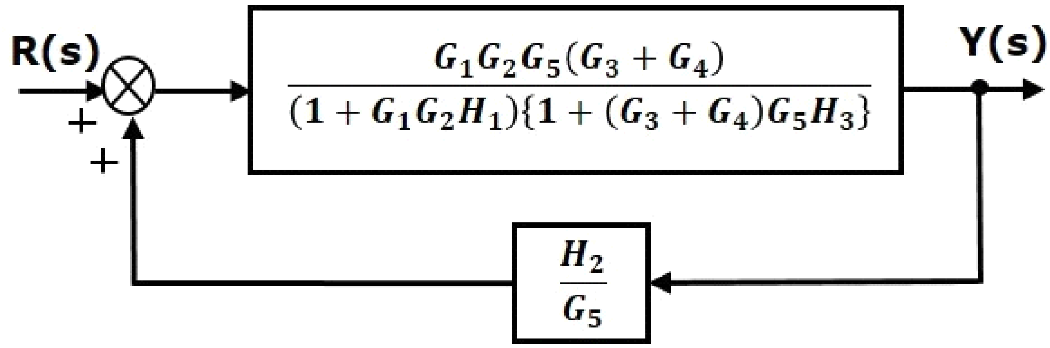
Step 3 – Use Rule 1 for blocks (G_3+G_4)(G3+G4) and G_5G5. The modified block diagram is shown in the following figure



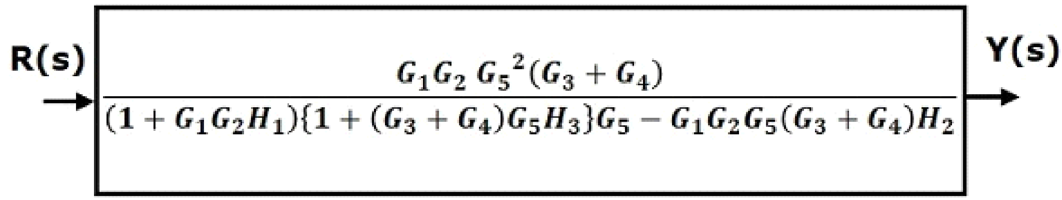
Step 4 – Use Rule 3 for blocks $(G_3+G_4)G_5$(G3+G4)G5 and H_3H3. The modified block diagram is shown in the following figure



Step 5 – Use Rule 1 for blocks connected in series. The modified block diagram is shown in the following figure



Step 6 – Use Rule 3 for blocks connected in feedback loop. The modified block diagram is shown in the following figure. This is the simplified block diagram.



Note – Follow these steps in order to calculate the transfer function of the block diagram having multiple inputs.

- **Step 1** – Find the transfer function of block diagram by considering one input at a time and make the remaining inputs as zero.
- **Step 2** – Repeat step 1 for remaining inputs.
- **Step 3** – Get the overall transfer function by adding all those transfer functions.

The block diagram reduction process takes more time for complicated systems. Because, we have to draw the (partially simplified) block diagram after each step. So, to overcome this drawback, use signal flow graphs (representation).

<u>Basic Elements of Signal Flow Graph</u>

Nodes and branches are the basic elements of signal flow graph.

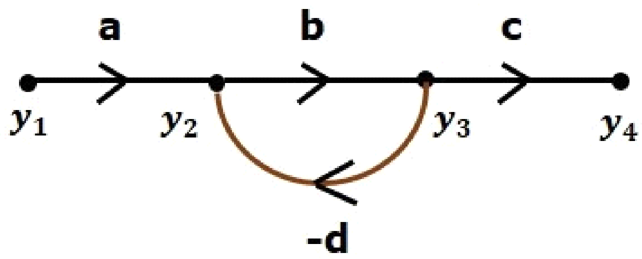
Node

Node is a point which represents either a variable or a signal. There are three types of nodes — input node, output node and mixed node.

- **Input Node** – It is a node, which has only outgoing branches.
- **Output Node** – It is a node, which has only incoming branches.
- **Mixed Node** – It is a node, which has both incoming and outgoing branches.

Example

Let us consider the following signal flow graph to identify these nodes.



- The **nodes** present in this signal flow graph are **y_1, y_2, y_3** and **y_4**.
- **y_1** and **y_4** are the **input node** and **output node** respectively.
- **y_2** and **y_3** are **mixed nodes**.

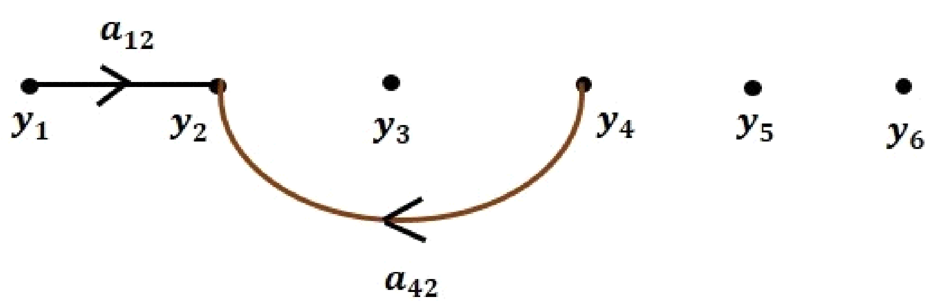
Branch

Branch is a line segment which joins two nodes. It has both **gain** and **direction**. For example, there are four branches in the above signal flow graph. These branches have **gains** of **a, b, c** and **-d**.

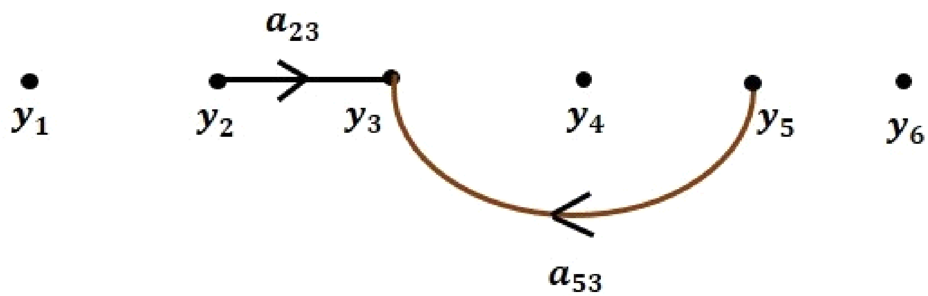
Construction of Signal Flow Graph

Let us construct a signal flow graph by considering the following algebraic equations –

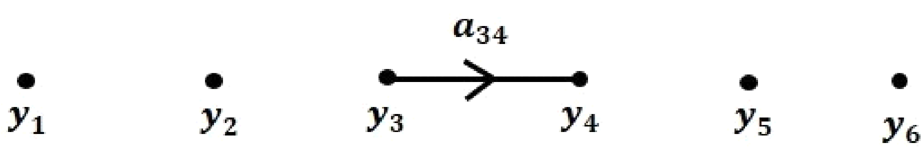
Step 1 – Signal flow graph for $y_2=a_{13}y_1+a_{42}y_4$y2=a13y1+a42y4 is shown in the following figure.



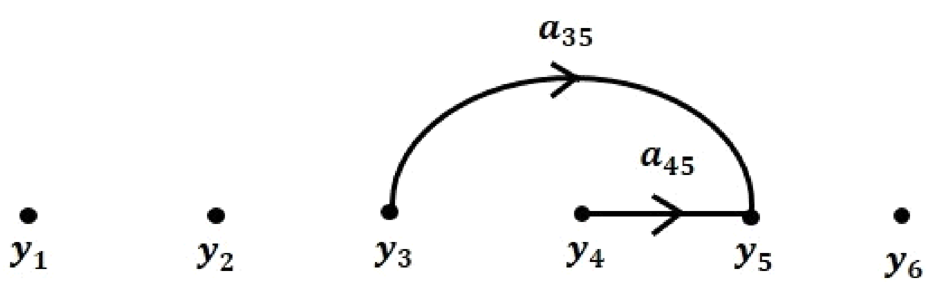
Step 2 – Signal flow graph for $y_3=a_{23}y_2+a_{53}y_5$y3=a23y2+a53y5 is shown in the following figure.



Step 3 – Signal flow graph for $y_4=a_{34}y_3$y4=a34y3 is shown in the following figure.

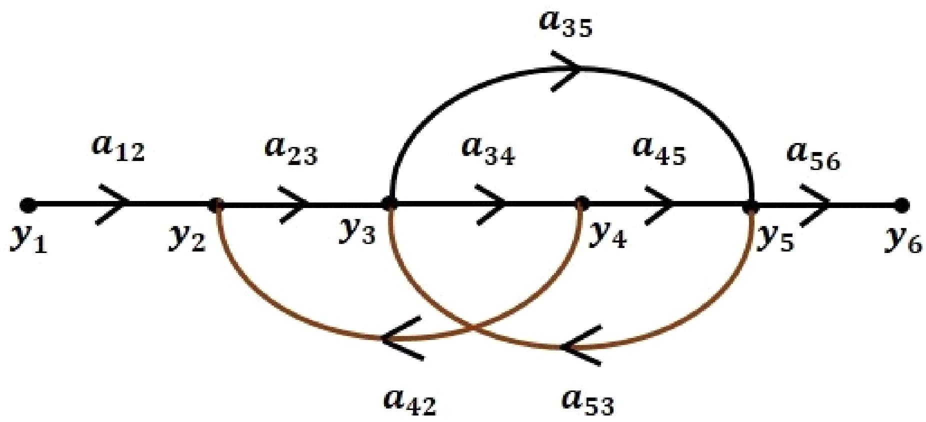


Step 4 – Signal flow graph for $y_5=a_{45}y_4+a_{35}y_3$y5=a45y4+a35y3 is shown in the following figure.



Step 5 – Signal flow graph for $y_6=a_{56}y_5$y6=a56y5 is shown in the following figure.

Step 6 – Signal flow graph of overall system is shown in the following figure.



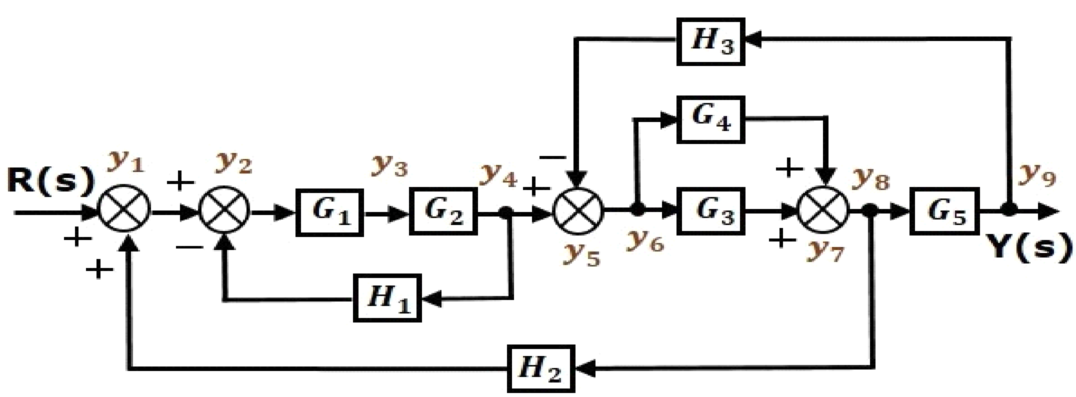
Conversion of Block Diagrams into Signal Flow Graphs

Follow these steps for converting a block diagram into its equivalent signal flow graph.

- Represent all the signals, variables, summing points and take-off points of block diagram as **nodes** in signal flow graph.
- Represent the blocks of block diagram as **branches** in signal flow graph.
- Represent the transfer functions inside the blocks of block diagram as **gains** of the branches in signal flow graph.
- Connect the nodes as per the block diagram. If there is connection between two nodes (but there is no block in between), then represent the gain of the branch as one. **For example**, between summing points, between summing point and takeoff point, between input and summing point, between take-off point and output.

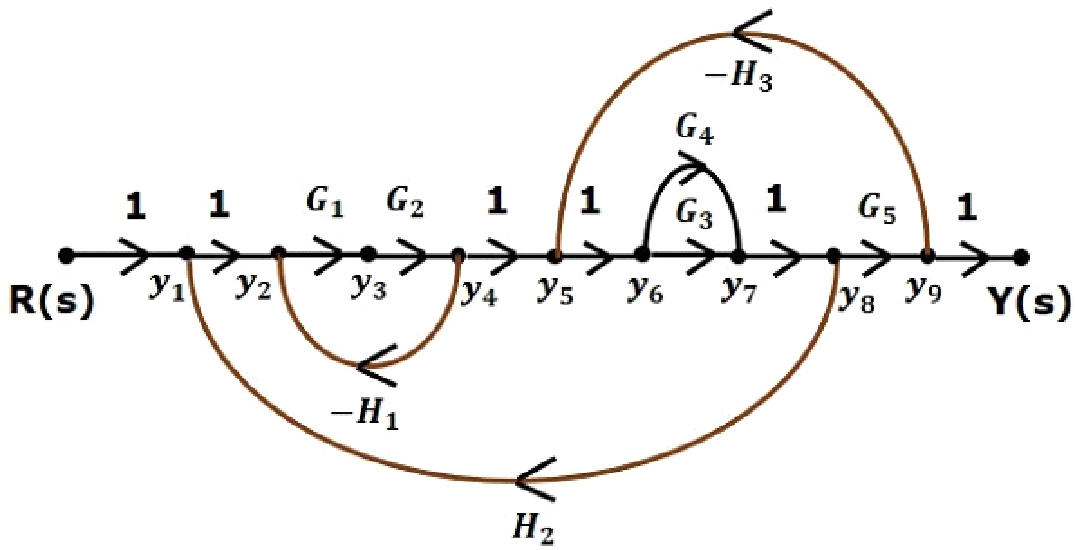
Example

Let us convert the following block diagram into its equivalent signal flow graph.



Represent the input signal R(s)R(s) and output signal C(s)C(s) of block diagram as input node R(s)R(s) and output node C(s)C(s) of signal flow graph.
Just for reference, the remaining nodes (y_1 to y_9) are labelled in the block diagram. There are nine nodes other than input and output nodes. That is four nodes for four summing points, four nodes for four take-off points and one node for the variable between blocks G1G1 and G2G2.

The following figure shows the equivalent signal flow graph.



With the help of Mason’s gain formula, you can calculate the transfer function of this signal flow graph. This is the advantage of signal flow graphs. Here, we no need to simplify (reduce) the signal flow graphs for calculating the transfer function.

Chapter 4 bcs

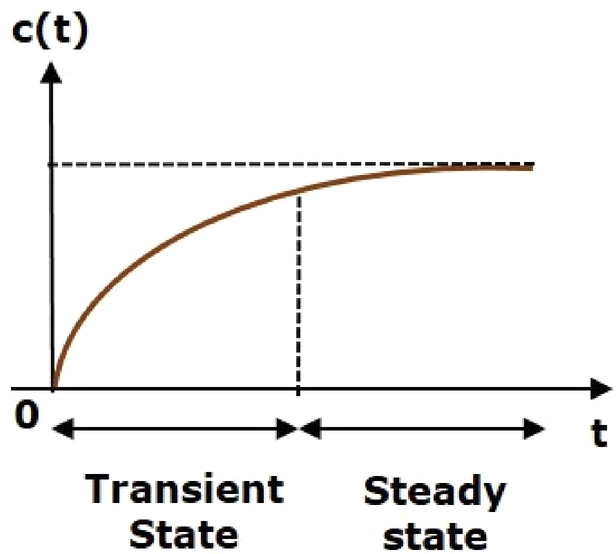
Time response analysis

What is Time Response

If the output of control system for an input varies with respect to time, then it is called the **time response** of the control system. The time response consists of two parts.

- Transient response
- Steady state response

The response of control system in time domain is shown in the following figure.



Here, both the transient and the steady states are indicated in the figure. The responses corresponding to these states are known as transient and steady state responses.

Mathematically, we can write the time response c(t) as

$$c(t)=c_{tr}(t)+c_{ss}(t)c(t)$$

Where,

- $c_{tr}(t)$ is the transient response
- $c_{ss}(t)$ is the steady state response

Transient Response

After applying input to the control system, output takes certain time to reach steady state. So, the output will be in transient state till it goes to a steady state. Therefore, the response of the control system during the transient state is known as **transient response**.

The transient response will be zero for large values of 't'. Ideally, this value of 't' is infinity and practically, it is five times constant.

Steady state Response

The part of the time response that remains even after the transient response has zero value for large values of 't' is known as **steady state response**. This means, the transient response will be zero even during the steady state.

Example

Let us find the transient and steady state terms of the time response of the control system $c(t)=10+5e^{-t}$c(t)=10+5e−t

Here, the second term $5e^{-t}$5e−t will be zero as **t** denotes infinity. So, this is the **transient term**. And the first term 10 remains even as **t** approaches infinity. So, this is the **steady state term**.

Standard Test Signals

The standard test signals are impulse, step, ramp and parabolic. These signals are used to know the performance of the control systems using time response of the output.

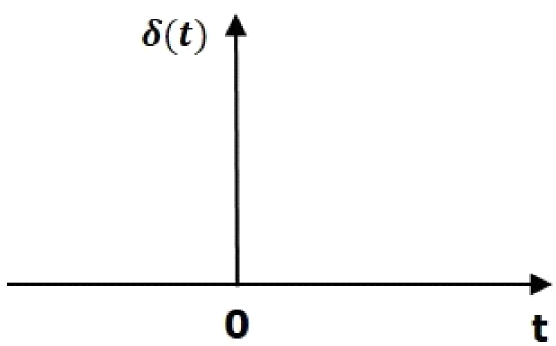
Unit Impulse Signal

A unit impulse signal, δ(t) is defined as

$\delta(t)=0$

δ(t)=0 for $t\neq0$

The following figure shows unit impulse signal.



So, the unit impulse signal exists only at 't' is equal to zero. The area of this signal under small interval of time around 't' is equal to zero is one. The value of unit impulse signal is zero for all other values of 't'.

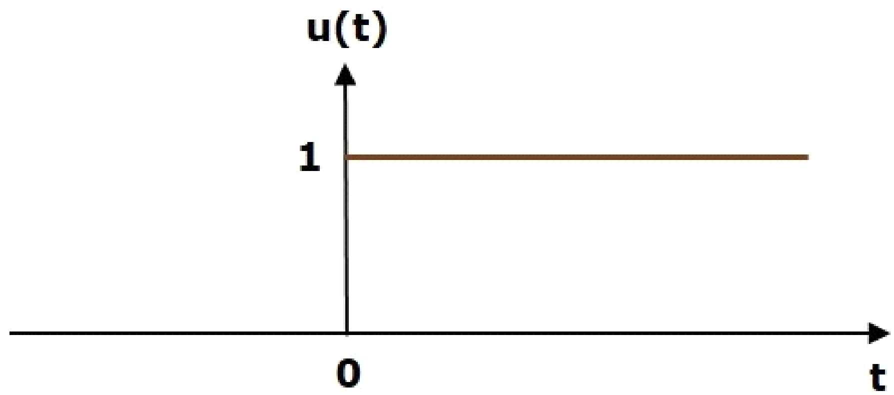
Unit Step Signal

A unit step signal, u(t) is defined as

$$u(t)=1 \quad ;t\geq 0$$

$$u(t)=0 \quad ;t<0$$

Following figure shows unit step signal.



So, the unit step signal exists for all positive values of ‘t’ including zero. And its value is one during this interval. The value of the unit step signal is zero for all negative values of ‘t’.

Unit Ramp Signal

A unit ramp signal, r(t) is defined as

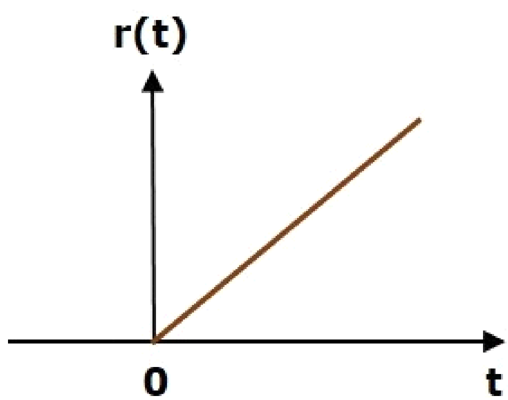
$$r(t)=t \quad ;t\geq 0$$

$$=0 \quad ;t<0$$

We can write unit ramp signal, $r(t)$ in terms of unit step signal, $u(t)$ as

$$r(t)=tu(t)$$

Following figure shows unit ramp signal.



So, the unit ramp signal exists for all positive values of ‘t’ including zero. And its value increases linearly with respect to ‘t’ during this interval. The value of unit ramp signal is zero for all negative values of ‘t’.

Unit Parabolic Signal

A unit parabolic signal, p(t) is defined as,

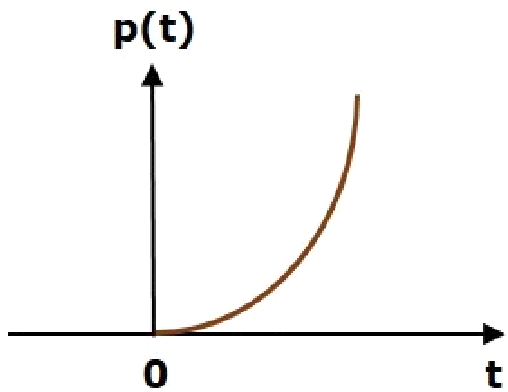
$$p(t)=t^2/2 \quad ;t\geq 0$$

$$=0 \quad ;t<0$$

We can write unit parabolic signal, $p(t)$ in terms of the unit step signal, $u(t)$ as,

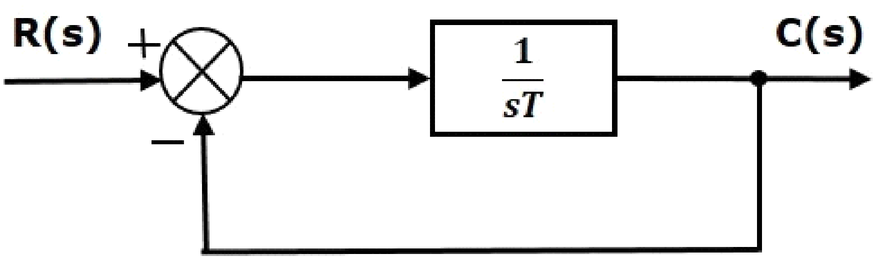
$p(t)=t2/2 \quad u(t)=t^2/2u(t)$

The following figure shows the unit parabolic signal.



So, the unit parabolic signal exists for all the positive values of **‘t’** including zero. And its value increases non-linearly with respect to ‘t’ during this interval. The value of the unit parabolic signal is zero for all the negative values of ‘t’.

In this chapter, let us discuss the time response of the first order system. Consider the following block diagram of the closed loop control system. Here, an open loop transfer function, 1sT1sT is connected with a unity negative feedback.



We know that the transfer function of the closed loop control system has unity negative feedback as,

=

Substitute, $G(s)=$ in the above equation.

$$1sT=1sT+1C(s)R(s)=1sT1+1sT=1sT+1$$

The power of s is one in the denominator term. Hence, the above transfer function is of the first order and the system is said to be the **first order system**.

We can re-write the above equation as

$$C(s)=(1sT+1)R(s)C(s)=(1sT+1)R(s)$$

Where,

- **C(s)** is the Laplace transform of the output signal c(t),
- **R(s)** is the Laplace transform of the input signal r(t), and
- **T** is the time constant.

Follow these steps to get the response (output) of the first order system in the time domain.

- Take the Laplace transform of the input signal $r(t)r(t)$.
- Consider the equation, $C(s)=(1sT+1)R(s)C(s)=(1sT+1)R(s)$
- Substitute $R(s)R(s)$ value in the above equation.
- Do partial fractions of $C(s)C(s)$ if required.
- Apply inverse Laplace transform to $C(s)C(s)$.

In the previous chapter, we have seen the standard test signals like impulse, step, ramp and parabolic. Let us now find out the responses of the first order system for each input, one by one. The name of the response is given as per the name of the input signal. For example, the response of the system for an impulse input is called as impulse response.

Impulse Response of First Order System

Consider the **unit impulse signal** as an input to the first order system.

So, $r(t)=\delta(t)r(t)=\delta(t)$

Apply Laplace transform on both the sides.

$R(s)=1R(s)=1$

Consider the equation, $C(s)=(1sT+1)R(s)C(s)=(1sT+1)R(s)$

Substitute, $R(s)=1R(s)=1$ in the above equation.

$$C(s)=(1sT+1)(1)=1sT+1C(s)=1sT+1$$

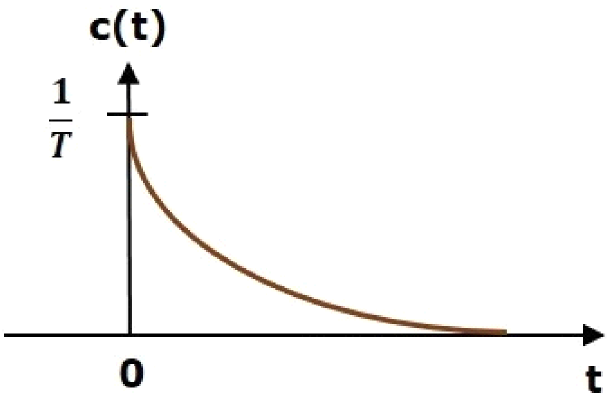
Rearrange the above equation in one of the standard forms of Laplace transforms.

$$C(s)=1T(s+1T)\Rightarrow C(s)=1T(1s+1T)C(s)=1T(s+1T)\Rightarrow C(s)=1T(1s+1T)$$

Apply inverse Laplace transform on both sides.

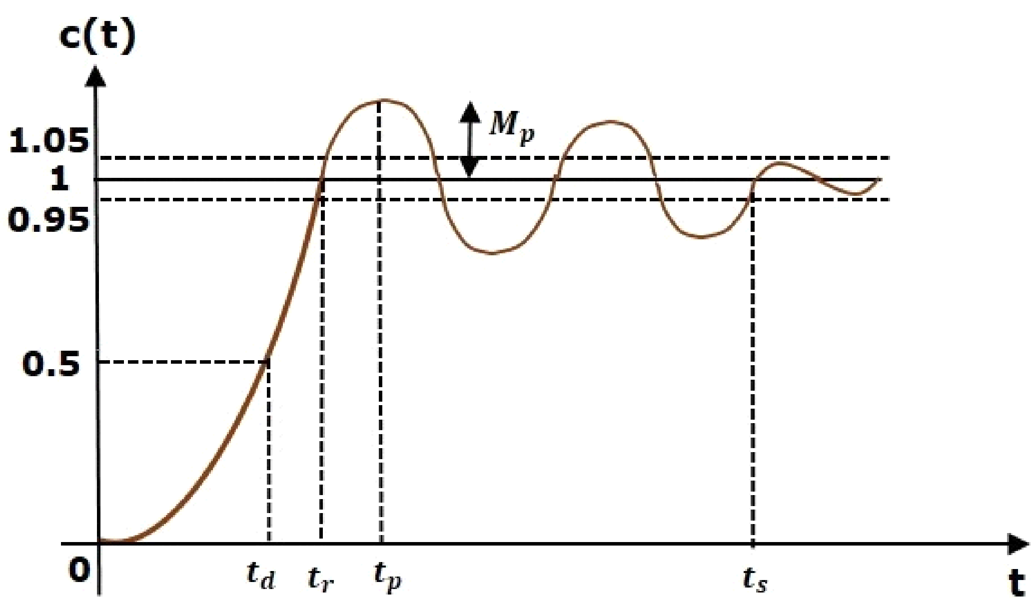
$$c(t)=1Te^{(-tT)}u(t)c(t)=1Te(-tT)u(t)$$

The unit impulse response is shown in the following figure.



The **unit impulse response**, c(t) is an exponential decaying signal for positive values of ‘t’ and it is zero for negative values of ‘t’.

<u>Time domain specifications :</u>



The step response of the second order system for the underdamped case is shown in the following figure.

All the time domain specifications are represented in this figure. The response up to the settling time is known as transient response and the response after the settling time is known as steady state response.

Delay Time

It is the time required for the response to reach **half of its final value** from the zero instant. It is denoted by t_d.

$$t_{d=}$$

Rise Time

It is the time required for the response to rise from **0% to 100% of its final value**. This is applicable for the **under-damped systems**. For the over-damped systems, consider the duration from 10% to 90% of the final value. Rise time is denoted by $\mathbf{t_r}$.

$$t_{r=}$$

Peak Time

It is the time required for the response to reach the **peak value** for the first time. It is denoted by tp. At $t=t_p t=tp$, the first derivate of the response is zero.

$$tp=$$

From the above equation, we can conclude that the peak time ttp and the damped frequency ω_d are inversely proportional to each other.

Peak Overshoot

Peak overshoot $\mathbf{M_p}$ is defined as the deviation of the response at peak time from the final value of response. It is also called the **maximum overshoot**.

Mathematically, we can write it as

$$Mp=e^{-()} \text{ X } 100$$

From the above equation, we can conclude that the percentage of peak will decrease if the damping ratio δ increases.

Settling time

It is the time required for the response to reach the steady state and stay within the specified tolerance bands around the final value. In general, the tolerance bands are 2% and 5%. The settling time is denoted by $t_s$ts.

The settling time for 5% tolerance band is -

$$t_s==3t$$

The settling time for 2% tolerance band is -

$$t_s = = 4t$$

Where, t is the time constant and is equal to $1\delta\omega n$.

- Both the settling time t_s and the time constant t are inversely proportional to the damping ratio δ.
- Both the settling time t_s and the time constant t are independent of the system gain. That means even the system gain changes, the settling time t_s and time constant t will never change.

Steady state error

- The deviation of the output of control system from desired response during steady state is known as **steady state error**. It is represented as e_{ss}. We can find steady state error using the final value theorem as follows.

  $$e_{s=} \quad =$$

  Where

  e(s) is the Laplace transform of the error signal, $e(t)$

Chapter 5 BCS

Routh Array's criterion:

Routh (1874) developed a a necessary and sufficient condition for stability based on Routh array, which states:

Routh's criterion: A system is stable if and only if all the elements in the first column of the Routh array are possitive.

Routh array: The first two rows of the Routh array are composed of the even and odd coefficients of the characteristic polynomial, respectively, while the remaining rows are composed of elements derived from the first two rows:

row n	a_0	a_2	a_4	$\cdots$
row n-1	a_1	a_3	a_5	$\cdots$
row n-2	b_1	a_2	a_3	$\cdots$
row n-3	c_1	b_2	b_3	$\cdots$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
row 2	$*$	$*$		$\cdots$
row 1	$*$			$\cdots$
row 0	$*$			$\cdots$

The elements of the third row are computed as follows:

$$b_1 = -\frac{det\begin{bmatrix} a_0 & a_2 \\ a_1 & a_3 \end{bmatrix}}{a_1} = \frac{a_1 a_2 - a_0 a_3}{a_1}$$

$$b_2 = -\frac{det\begin{bmatrix} a_0 & a_4 \\ a_1 & a_5 \end{bmatrix}}{a_1} = \frac{a_1 a_4 - a_0 a_5}{a_1}$$

$$b_3 = -\frac{det\begin{bmatrix} a_0 & a_6 \\ a_1 & a_7 \end{bmatrix}}{a_1} = \frac{a_1 a_6 - a_0 a_7}{a_1}$$

The elements of the forth row are computed as follows:

$$c_1 = -\frac{det\begin{bmatrix} a_1 & a_3 \\ b_1 & b_2 \end{bmatrix}}{b_1} = \frac{b_1 a_3 - a_1 b_2}{b_1}$$

$$c_2 = -\frac{det\begin{bmatrix} a_1 & a_5 \\ b_1 & b_3 \end{bmatrix}}{b_1} = \frac{b_1 a_5 - a_1 b_3}{b_1}$$

$$c_3 = -\frac{det\begin{bmatrix} a_1 & a_7 \\ b_1 & b_4 \end{bmatrix}}{b_1} = \frac{b_1 a_7 - a_1 b_4}{b_1}$$

The elements from the third row on are computed based on the determinant of a 2 by 2 array composed of the two elements of the first column of the previous two rows and the two elements of the subsequent columns. Any missing coefficient is represented by a zero.

Example 0:

$$D(s) = a_0 s^3 + a_1 s^2 + a_2 s + a_3 = 0$$

$$\begin{array}{lrl} \text{row } 3 & a_0 & a_2 \\ \text{row } 2 & a_1 & a_3 \\ \text{row } 1 & (a_2 a_1 - a_0 a_3)/a_1 & \\ \text{row } 0 & a_3 & \end{array}$$

For this system to be stable, we must have $a_2a_1 - a_0a_3 > 0$.

Example 1:

$$D(s) = s^4 + 2s^3 + 3s^2 + 4s + 5 = 0$$

$$\begin{array}{rrrr}
\text{row } 4 & 1 & 3 & 5 \\
\text{row } 3 & 2 & 4 & 0 \\
\text{row } 3' & 1 & 2 & 0 \\
\text{row } 2 & 1 & 5 & \\
\text{row } 1 & -3 & & \\
\text{row } 0 & 5 & &
\end{array}$$

There are two sign changes indicating two poles on RP. Note that row 3 is divided by 2 to become row 3' without affecting the result.

Example 2:

$$D(s) = s^6 + 4s^5 + 3s^4 + 2s^3 + s^2 + 4s + 4$$

with $a_0 = 1, a_1 = 4, a_2 = 3, a_3 = 2, a_4 = 1, a_5 = 4, a_6 = 4$. First we find the Routh array:

$$\begin{array}{lrrrr}
\text{row } 6 & 1 & 3 & 1 & 4 \\
\text{row } 5 & 4 & 2 & 4 & 0 \\
\text{row } 4 & (4\cdot 3 - 1\dot{2})/4 = 2.5 & (4\cdot 1 - 4\dot{1})/4 = 0 & (4\cdot 4 - 1\cdot 0)/4 = 4 & \\
\text{row } 3 & (2.5\cdot 2 - 4\cdot 0)/2.5 = 2 & (2.5\cdot 4 - 4\cdot 4)/2.5 = -2.4 & 0 & \\
\text{row } 2 & (2\cdot 0 - 2.5\cdot(-2.4))/2 = 3 & (2\cdot 4 - 2.5\cdot 0) = 4 & 0 & \\
\text{row } 1 & (3\cdot(-2.4) - 2\cdot 4)/3 = -76/15 & & & \\
\text{row } 0 & (-76/15\cdot 4 - 0\cdot 3)/(-76/15) = 4 & & &
\end{array}$$

The elements in the first column are: 1, 4, 2.5, 2, 3, -76/15, 4 with two sign changes (3 to -76/15 and -76/15 to 4), there are two poles on the RP and the system is not stable. Solving the characteristic equation, we can get the five roots: $-3.26, 0.68 \pm 0.75i, -0.60 \pm 0.99i, -0.89$.

Example 3:

The transfer function of the feedforward pass of a feedback system is

$$H(s) = K\,\frac{s+1}{s(s-1)(s+6)}$$

and the feedback gain is G(s) = - 1 (negative feedback). The overall transfer function is therefore:

$$T(s) = \frac{H(s)}{1-H(s)G(s)} = \frac{K\,\frac{s+1}{s(s+1)(s+2)}}{1+\frac{s+1}{s(s+1)(s+2)}} = \frac{K(s+1)}{s(s-1)(s+6)+K(s+1)}$$

The characteristic equation is

$$s^3 + 5s^2 + (K-6)s + K = 0$$

Routh's criterion is used To find the range of the gain K for stability:

$$\begin{array}{lrr} \text{row } 3 & 1 & K-6 \\ \text{row } 2 & 5 & K \\ \text{row } 1 & (4K-30)/5 & 0 \\ \text{row } 0 & K & 0 \end{array}$$

For all elements of the first column to be positive, we have $K > 0$ and $K > 7.5$. Consider the following three cases:

- When K=9 > 7.5, the roots of the characteristic equation are P_0 = -4.77, P1.2 = -0.17 $\pm$ 1.37 , representing respectively an exponentially decaying term and a sinusoid also exponentially decaying.
- When $K = 7.5$, the characteristic equation is

  $$D(s) = s^3+5s^2+1.5s+7.5 = (s+5)(s^2+1.5) = (s+5)(s+j\sqrt{1.5})(s-j\sqrt{1.5}) = 0$$

  with three poles P_0 = -5 representing an exponential decay and a complex conjugate pair $P_{1.2} = \pm\, i\sqrt{1.5}$ representing a sinusoidal oscillation.

When K = 5 < 7.5, the roots of the characteristic equation are P_0 = -5.36, $P_{1.2}$ = 0.18 $\pm$ 095i , representing respectively an exponentially decaying **Special Case 1:**

If the first column contains a zero, the elements in the following row cannot be evaluated (divided by zero). In this case, the zero will be replaced by a small value $\epsilon > 0$ and the elements of the following rows can be obtained in terms of ϵ. At the end, we take the limit $\epsilon \rightarrow 0$.

Example 6:

$$D(s) = s^5 + 3s^4 + 2s^3 + 6s^2 + 6s + 9 = 0$$

$$\begin{array}{lrrr}
\text{row } 5 & 1 & 2 & 6 \\
\text{row } 4 & 3 & 6 & 9 \\
\text{row } 4' & 1 & 2 & 3 \\
\text{row } 3 & 0 & 3 & \\
\text{row } 3' & \epsilon & 3 & \\
\text{row } 2 & (2\epsilon - 3)/\epsilon & 3 & \\
\text{row } 1 & 3 - 3\epsilon^2/(2\epsilon - 3) & 0 & \\
\text{row } 0 & 3 & 0 &
\end{array}$$

Taking the limit $\epsilon \rightarrow 0$, the first element in row 2 is negative, while the first element in row 1 is positive, i.e., there are two sign changes in the first column indicating there are two roots on the RP. In fact, the five roots are
$-2.9, 0.66 \pm 1.29i, -0.70 \pm 0.99$

.

Special Case 2:

It is possible that all elements of a row, assumed to be the ith row, are zero. In such case, the rest of the Routh array can still be obtained by the following steps:

- treat the previous (i+1)th row as an auxiliary polynomial P(s)
- take derivative of P(s), and use the resulting coefficients as the elements of the ith row
- continue as normal to obtain the rest elements of the array.

Example 1:

$$D(s) = s^5 + 2s^4 + 24s^3 + 48s^2 - 25s - 50 = 0$$

row 5	1	24	-25
row 4	2	48	-50
row 4'	1	24	-25
row 3	0	0	
row 3'	4	48	
row 3"	1	12	
row 2	24	-50	
row 1	14.08	0	
row 0	-50		

As all elements in row 3 are zero, an auxiliary polynomial is formed using the elements of the previoius row and its derivative is obtained:

$$P(s) = s^4 + 24s^2 - 25, \qquad \frac{d}{ds}P(s) = 4s^3 + 48s$$

The coefficients of P (s) are then used as the elements of the ith row and the process continues as normal. As there is only one sign change in the first column, there is one root on the RP.

The auxiliary polynomial P(s) = $s^4 + 24s^2 - 25 = (s^2 - 1)(s^2 + 25)$ can be solved to find its roots s =±1 and s =±i5 . They always lie radially opposite in the S-plane. the original characteristic polynomial can be factored as

$$D(s) = s^5 + 2s^4 + 24s^3 + 48s^2 - 25s - 50 = (s^4 + 24s^2 - 25)(s + 2) = 0$$

It can be seen that the roots of D (s) include those of D (s), with an additional one s= -2. term and a sinusoid which grows exponentially.

Root locus

The root locus of a feedback system is the graphical representation in the complex s-plane of the possible locations of its closed-loop poles for varying values of a certain system parameter. The points that are part of the root locus satisfy the angle condition.

The root locus technique in control system was first introduced in the year 1948 by Evans. Any physical system is represented by a transfer function in the form of

$$G(s) = k \times \frac{numerator\ of\ s}{denomerator\ of\ s}$$

We can find poles and zeros from G(s). The location of poles and zeros are crucial keeping view stability, relative stability, transient response and error analysis. When the system is put to service stray inductance and capacitance get into the system, thus changes the location of poles and zeros. In root locus technique in control system we will evaluate the position of the roots, their locus of movement and associated information. These information will be used to comment upon the system performance.

Now before I introduce what is a root locus technique, it is very essential here to discuss a few of the advantages of this technique over other stability criteria. Some of the advantages of root locus technique are written below.

Advantages of Root Locus Technique

- Root locus technique in control system is easy to implement as compared to other methods.
- With the help of root locus we can easily predict the performance of the whole system.
- Root locus provides the better way to indicate the parameters.

Now there are various terms related to root locus technique that we will use frequently in this article.

- Characteristic Equation Related to Root Locus Technique : 1 + G(s)H(s) = 0 is known as characteristic equation. Now on differentiating the characteristic equation and on equating dk/ds equals to zero, we can get break away points.
- Break away Points : Suppose two root loci which start from pole and moves in opposite direction collide with each other such that after collision they start moving in different directions in the symmetrical way. Or the breakaway points at which multiple roots of the characteristic equation 1 + G(s)H(s) = 0 occur. The value of K is maximum at the points where the branches of root loci break away. Break away points may be real, imaginary or complex.
- Break in Point : Condition of break in to be there on the plot is written below : Root locus must be present between two adjacent zeros on the real axis.
- Centre of Gravity : It is also known centroid and is defined as the point on the plot from where all the asymptotes start. Mathematically, it is calculated by the difference of summation of poles and zeros in the transfer function when divided by the difference of total number of poles and total number of zeros.

Centre of gravity is always real and it is denoted by σ_A.

$$\sigma_A = \frac{(Sum\ of\ real\ parts\ of\ poles) - (Sum\ of\ real\ parts\ of\ zeros)}{N - M}$$

Where, N is number of poles and M is number of zeros.

- Asymptotes of Root Loci : Asymptote originates from the center of gravity or centroid and goes to infinity at definite some angle. Asymptotes provide direction to the root locus when they depart break away points.
-
- Angle of Asymptotes : Asymptotes makes some angle with the real axis and this angle can be calculated from the given formula,

$$Angle\ of\ asymptotes\ = \frac{(2p+1)\times 180}{N-M}$$

Where, p = 0, 1, 2 ……. (N-M-1) N is the total number of poles M is the total number of zeros.

- Angle of Arrival or Departure : We calculate angle of departure when there exists complex poles in the system. Angle of departure can be calculated as 180-{(sum of angles to a complex pole from the other poles)-(sum of angle to a complex pole from the zeros)}.

- Intersection of Root Locus with the Imaginary Axis : In order to find out the point of intersection root locus with imaginary axis, we have to use Routh Hurwitz criterion. First, we find the auxiliary equation then the corresponding value of K will give the value of the point of intersection.

- Gain Margin : We define gain margin by which the design value of the gain factor can be multiplied before the system becomes unstable. Mathematically it is given by the formula

$$Gain\ margin = \frac{Value\ of\ K\ at\ the\ imaginary\ axes\ cross\ over}{Design\ value\ of\ K}$$

- Phase Margin : Phase margin can be calculated from the given formula:

$$Phase\ margin = 180 + \angle(G(jw)H(jw))$$

- Symmetry of Root Locus : Root locus is symmetric about the x axis or the real axis.

How to determine the value of K at any point on the root loci? Now there are two ways of determining the value of K, each way is described below.

- Magnitude Criteria: At any points on the root locus we can apply magnitude criteria as,

$$|G(s)H(s)| = 1$$

Using this formula we can calculate the value of K at any desired point.

- Using Root Locus Plot : The value of K at any s on the root locus is given by

$$K = \frac{product\ of\ all\ of\ the\ vector\ lengths\ drawn\ from\ the\ poles\ of\ G(s)H(s)\ to\ s}{product\ of\ all\ of\ the\ vector\ lengths\ drawn\ from\ the\ zeros\ of\ G(s)H(s)\ to\ s}$$

Root Locus Plot

This is also known as root locus technique in control system and is used for determining the stability of the given system. Now in order to determine the stability of the system using the root locus technique we find the range of values of K for which the complete performance of the system will be satisfactory and the operation is stable.
Now there are some results that one should remember in order to plot the root locus. These results are written below:

- Region where root locus exists : After plotting all the poles and zeros on the plane, we can easily find out the region of existence of the root locus by using one simple rule which is written below,
  Only that segment will be considered in making root locus if the total number of poles and zeros at the right hand side of the segment is odd.
- How to calculate the number of separate root loci ? : A number of separate root loci are equal to the total number of roots if number of roots are greater than the number of poles otherwise number of separate root loci is equal to the total number of poles if number of roots are greater than the number of zeros.

Procedure to Plot Root Locus

Keeping all these points in mind we are able to draw the **root locus plot** for any kind of system. Now let us discuss the procedure of making a root locus.

- Find out all the roots and poles from the open loop transfer function and then plot them on the complex plane.
- All the root loci starts from the poles where k = 0 and terminates at the zeros where K tends to infinity. The number of branches terminating at infinity equals to the difference between the number of poles & number of zeros of G(s)H(s).
- Find the region of existence of the root loci from the method described above after finding the values of M and N.
- Calculate break away points and break in points if any.
- Plot the asymptotes and centroid point on the complex plane for the root loci by calculating the slope of the asymptotes.
- Now calculate angle of departure and the intersection of root loci with imaginary axis.
- Now determine the value of K by using any one method that I have described above.

By following above procedure you can easily draw the **root locus plot** for any open loop transfer function.

- Calculate the gain margin.
- Calculate the phase margin.
- You can easily comment on the stability of the system by using Routh Array.

Bode Plot

A **Bode plot** is a graph commonly used in control system engineering to determine the stability of a control system. A Bode plot maps the frequency response of the system through two graphs – the **Bode magnitude plot** (expressing the magnitude in decibels) and the **Bode phase plot** (expressing the phase shift in degrees).
Bode plots were first introduced in the 1930s by Hendrik Wade Bode while he was working at Bell Labs in the United States. Although Bode plots offer a relatively simple method to calculate system stability, they can not handle transfer functions with right half plane singularities.
Understanding gain margins and phase margins is crucial to understanding Bode plots. These terms are defined below.

Gain Margin

The greater the Gain Margin (GM), the greater the stability of the system. The gain margin refers to the amount of gain, which can be increased or decreased without making the system unstable. It is usually expressed as a magnitude in dB.
We can usually read the gain margin directly from the Bode plot (as shown in the diagram above). This is done by calculating the vertical distance between the magnitude curve (on the Bode magnitude plot) and the x-axis at the frequency where the Bode phase plot = 180°. This point is known as the phase crossover frequency.

Phase Margin

The greater the **Phase Margin** (PM), the greater will be the stability of the system. The phase margin refers to the amount of phase, which can be increased or decreased without making the system unstable. It is usually expressed as a phase in degrees.
We can usually read the phase margin directly from the Bode plot (as shown in the diagram above). This is done by calculating the vertical distance between the phase curve (on the Bode phase plot) and the x-axis at the frequency where the Bode magnitude plot = 0 dB. This point is known as the **gain crossover frequency**.

Bode Plot Stability

Below are a summarised list of criterion relevant to drawing Bode plots (and calculating their stability):

- Gain Margin: Greater will the gain margin greater will be the stability of the system. It refers to the amount of gain, which can be increased or decreased without making the system unstable. It is usually expressed in dB.
- Phase Margin: Greater will the phase margin greater will be the stability of the system. It refers to the phase which can be increased or decreased without making the system unstable. It is usually expressed in phase.
- Gain Crossover Frequency: It refers to the frequency at which magnitude curve cuts the zero dB axis in the bode plot.
- Phase Crossover Frequency: It refers to the frequency at which phase curve cuts the negative times the 180^{o} axis in this plot.
- Corner Frequency: The frequency at which the two asymptotes cuts or meet each other is known as break frequency or corner frequency.
- Resonant Frequency: The value of frequency at which the modulus of G (jω) has a peak value is known as the resonant frequency.
- Factors: Every loop transfer function {i.e. G(s) × H(s)} product of various factors like constant term K, Integral factors (jω), first-order factors (1 + jωT)$^{(\pm n)}$ where n is an integer, second order or quadratic factors.
- Slope: There is a slope corresponding to each factor and slope for each factor is expressed in the dB per decade.
- Angle: There is an angle corresponding to each factor and angle for each factor is expressed in the degrees.

Now there are some results that one should remember in order to plot the Bode curve. These results are written below:

- Constant term K: This factor has a slope of zero dB per decade. There is no corner frequency corresponding to this constant term. The phase angle associated with this constant term is also zero.
- Integral factor $1/(j\omega)^n$: This factor has a slope of -20 × n (where n is any integer)dB per decade. There is no corner frequency corresponding to this integral factor. The phase angle associated with this integral factor is -90 × n here n is also an integer.
- First order factor $1/(1+j\omega T)$: This factor has a slope of -20 dB per decade. The corner frequency corresponding to this factor is 1/T radian per second. The phase angle associated with this first factor is $-\tan^{-1}(\omega T)$.
- First order factor $(1+j\omega T)$: This factor has a slope of 20 dB per decade. The corner frequency corresponding to this factor is 1/T radian per second. The phase angle associated with this first factor is $\tan^{-1}(\omega T)$.
- Second order or quadratic factor : $[\{1/(1+(2\zeta/\omega)\} \times (j\omega) + \{(1/\omega^2)\} \times (j\omega)^2)]$: This factor has a slope of -40 dB per decade. The corner frequency corresponding to this factor is ω^n radian per second. The phase angle associated with this first factor is $tan^{-1}\left\{\frac{\frac{2\zeta\omega}{\omega n}}{1-\left(\frac{\omega}{\omega n}\right)^2}\right\}$

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The Gain Margin and Phase Margin shown on a Bode Plot

How to Draw Bode Plot

Keeping all the above points in mind, we are able to draw a Bode plot for any kind of control system. Now let us discuss the procedure of drawing a Bode plot:

- Substitute the $s = j\omega$ in the open loop transfer function $G(s) \times H(s)$.
- Find the corresponding corner frequencies and tabulate them.
- Now we are required one semi-log graph chooses a frequency range such that the plot should start with the frequency which is lower than the lowest corner frequency. Mark angular frequencies on the x-axis, mark slopes on the left hand side of the y-axis by marking a zero slope in the middle and on the right hand side mark phase angle by taking -180^{o} in the middle.
- Calculate the gain factor and the type or order of the system.
- Now calculate slope corresponding to each factor.

For drawing the **Bode magnitude plot**:

- Mark the corner frequency on the semi-log graph paper.
- Tabulate these factors moving from top to bottom in the given sequence.
  - Constant term K.

- Integral factor $\frac{1}{j\omega^n}$
- First order factor $\frac{1}{1+j\omega T}$
- First order factor (1+jωT).
- Second order or quadratic factor: $\left[\frac{1}{1+(2\zeta/\omega)} \times (j\omega) + \left(\frac{1}{\omega^2}\right) \times (j\omega)^2\right]$

- Now sketch the line with the help of the corresponding slope of the given factor. Change the slope at every corner frequency by adding the slope of the next factor. You will get the magnitude plot.
- Calculate the gain margin.

For drawing the **Bode phase plot**:

- Calculate the phase function adding all the phases of factors.
- Substitute various values to the above function in order to find out the phase at different points and plot a curve. You will get a phase curve.
- Calculate the phase margin.

Bode Stability Criterion

Stability conditions are given below:

- For Stable System: Both the margins should be positive or phase margin should be greater than the gain margin.
- For Marginal Stable System: Both the margins should be zero or phase margin should be equal to the gain margin.

- For Unstable System: If any of them is negative or **phase margin** should be less than the gain margin.

Advantages of Bode Plot

- It is based on the asymptotic approximation, which provides a simple method to plot the logarithmic magnitude curve.
- The multiplication of various magnitude appears in the transfer function can be treated as an addition, while division can be treated as subtraction as we are using a logarithmic scale.
- With the help of this plot only we can directly comment on the stability of the system without doing any calculations.
- **Bode plots** provides relative stability in terms of **gain margin** and **phase margin**.
- It also covers from low frequency to high frequency range.